Beyond Mass-Energy Equivalence: A Category-Theoretic Exploration of Incommensurable Physical Realities

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This paper explores a speculative yet formally grounded thought experiment: what if two equally intelligent civilizations develop entirely different frameworks for physics—each internally coherent, predictive, and experimentally verified—yet conceptually unintelligible to the other? One uses the familiar human formulation of mass-energy equivalence, E = mc^2, rooted in classical spacetime and energetic quantities. The other, however, describes the same physical transformation using different primitives that are semantically inaccessible to us. While no direct translation of terms is possible, both civilizations converge on structurally equivalent laws. To reconcile this paradox, we argue for a shift from semantic to structural interpretation of physical laws. Using the tools of Category Theory and Homotopy Type Theory (HoTT), we demonstrate how structurally invariant formulations can exist independently of conceptual overlap. This reframing has implications not only for our understanding of physics but also for SETI, interspecies communication, and the foundations of scientific universality. We contend that what is truly universal may not be the content of scientific laws, but the abstract, relational structure they instantiate—a structure intelligible across epistemic boundaries, even if their meaning is not.

Keywords: mass-energy equivalence, category theory, homotopy type theory, structural invariance, incommensurable theories, alien physics, interspecies epistemology, universal laws, topoi, functorial mapping, scientific translation, cognitive divergence, physical law, SETI, meta-mathematics, natural transformations, foundational physics, Gödel encoding, Turing equivalence, commutative diagrams. A collaboration with GPT-4o. CC4.0.

Abstract

The foundational equation of modern physics, E = mc^2, expresses a profound truth about the equivalence of mass and energy within the framework of human-defined concepts such as mass, energy, and the invariant speed of light. However, this formulation is rooted in cognitive and perceptual categories that may not generalize beyond human experience. In this paper, we present a thought experiment in which an alien civilization, equal in intelligence and scientific sophistication, has developed its own physics entirely detached from human ontologies. Though their system lacks any notion of mass, energy, or light, it nevertheless contains an invariant transformation law that is structurally equivalent to our mass-energy equivalence—but expressed in completely different primitives. Remarkably, while neither civilization can comprehend the other's physical theory in semantic terms, both are capable of understanding the *structure* of each other's theories through shared formal languages—specifically, Category Theory and Homotopy Type Theory (HoTT).

We argue that these foundational mathematical frameworks enable an abstract yet precise correspondence between seemingly incommensurable physical models by preserving higher-order relations rather than the constituent terms themselves. Through this lens, physical truth is not tied to specific representational content but emerges from structural invariants across different conceptual systems. This paper explores how such mappings are possible, the limits of inter-theoretical translation, and the broader implications for the universality of physical law. Ultimately, we suggest that Category Theory and HoTT constitute a kind of meta-physics: a formal substrate in which all intelligible physics—human or otherwise—can be embedded, compared, and understood, albeit in ways that transcend human intuition.

1. Introduction

Since the dawn of modern science, physicists have sought to uncover laws that describe the universe in universal, objective terms—laws that are presumed to

hold regardless of the observer's cultural background, biological form, or cognitive architecture. Central to this pursuit is the idea that certain mathematical formulations are not merely human inventions, but expressions of deeper truths about reality itself. One such formulation is Einstein's equation E = mc^2, a concise and elegant representation of the equivalence between mass and energy. Within the framework of human physics, it holds a position of extraordinary explanatory and predictive power, underpinning everything from nuclear energy to the life cycles of stars.

But is this equation, and the physics it arises from, truly universal? Or is it a contingent articulation shaped by the peculiarities of human cognition—our sensory apparatus, our neurobiological wiring, our linguistic habits, and our historical development of mathematical tools? To explore this question, we pose a thought experiment: imagine an alien civilization as intelligent as ours, yet possessing an entirely different set of cognitive and perceptual modalities. Their physics, shaped by their own sensory and conceptual frameworks, lacks any notions of "mass," "energy," or "the speed of light." And yet, within their own formal system, they have developed an equation that plays the same structural role as E = mc^2—governing the same class of transformational processes in their reality, albeit through different primitives and conceptual anchors.

This scenario raises a fundamental paradox. On one hand, the two civilizations are describing the same physical processes. They can both predict, measure, and manipulate transformations in their respective domains that, when compared through observational outcomes, appear identical. On the other hand, they are entirely unable to comprehend each other's physical theories in terms of meaning. Their concepts, definitions, and intuitions are so alien that direct translation is impossible. This paradox highlights the potential limitations of semantic understanding in physics and points to the possibility that what we call "physical law" may be better understood as an expression of deeper, more abstract structural relationships.

To resolve this paradox, we propose a shift in perspective—from substance to structure, from semantics to syntax. Specifically, we argue that Category Theory

and Homotopy Type Theory (HoTT) provide a shared meta-mathematical framework that allows these otherwise incommensurable physical theories to be mapped onto each other at the level of structure. These frameworks do not rely on the specific content of a theory's primitives but instead model the relationships between entities and transformations abstractly. In doing so, they offer a way to understand how different civilizations might arrive at the same physical truth through incompatible, irreconcilable representations.

This paper explores this idea in depth. We begin by elaborating on the thought experiment and laying out the terms of the paradox. We then introduce the foundational concepts of Category Theory and HoTT, showing how these tools can formalize equivalence between physical theories without requiring conceptual overlap. Finally, we examine the broader implications for how we think about physical law, scientific communication, and the universality of mathematics in a cosmically pluralistic context.

2. The Thought Experiment

To explore the limits of scientific universality and the potential role of abstract structure in bridging otherwise incommensurable worldviews, we propose a thought experiment involving two advanced civilizations. Both are equally intelligent, equally technologically capable, and equally committed to understanding the fabric of their respective realities. However, they are radically divergent in terms of cognition, perception, and conceptual frameworks.

The first civilization—ours—is grounded in a physics shaped by human sensory experience and historical development. We define physical quantities such as *mass*, *energy*, and *the speed of light* based on empirical observations that we interpret through the lens of human-intelligible constructs. Our physical theories are embedded within this ontology, and central among them is the Einsteinian formulation E = mc^2, which elegantly captures the relationship between mass and energy, mediated by the constant ccc, the speed of light in vacuum. This

equation plays a foundational role in our understanding of everything from subatomic particle behavior to astrophysical processes.

Now consider a second civilization that has evolved under entirely different conditions. Their perceptual apparatus is dissimilar to ours—not limited to the visible spectrum, perhaps not even based on sensory modalities we would recognize as such. Their minds operate according to a different cognitive architecture, one that structures experience in a way that does not naturally yield concepts like “mass” or “energy.” Instead, their scientific framework is built upon a distinct set of ontological and epistemological primitives—concepts that have no direct analogues in human thought.

Despite this, their scientists have also discovered a transformation law that governs a conserved invariant across all physical interactions in their universe. This law has predictive power, empirical validation, and functional utility in the same way that E = mc^2 does for us. But crucially, the primitives in their equation—let us call them Ξ, Φ, and χ, —do not map in any meaningful way onto human concepts. No amount of translation, explanation, or education allows one civilization to *understand* the other's formulation in a semantic sense. Even shared mathematical tools such as tensors, differential geometry, or analysis do not suffice to bridge the gap, because these tools themselves are implemented through different cognitive pathways and applied to different foundational concepts.

What, then, can be said about the relationship between the two civilizations’ physical theories? Clearly, they are not equivalent in content—but they are functionally equivalent in structure. Each system describes the same set of empirical invariants and transformation laws within its own conceptual universe. From the standpoint of an external observer—say, an omniscient intelligence with access to both systems—one might say that both civilizations have arrived at *structurally identical* models of physical reality, despite employing entirely different conceptual tools.

This scenario challenges our assumptions about what it means for a scientific theory to be "true" or "understood." It suggests that truth in physics may not reside in the specific content of a theory—the meaning of "mass" or "energy"—but in the formal relationships between entities and the transformations that govern them. It also underscores the limits of translation when it comes to scientific theories: even if two civilizations observe the same events and predict the same outcomes, their ways of organizing and conceptualizing those experiences may be mutually incomprehensible.

The implications for interspecies epistemology are profound. If human beings ever encounter another intelligent species—or an artificial intelligence whose cognitive architecture diverges from our own—we may find ourselves unable to communicate using the content of our science. Instead, we may need to rely on shared formal structures, such as those found in Category Theory and Homotopy Type Theory, to establish a framework for understanding. In doing so, we may come to view our own theories not as ultimate truths, but as one of many possible realizations of a deeper, more abstract order.

3. Category Theory as a Universal Structural Language

If two civilizations possess fundamentally different conceptual frameworks for understanding physical reality, yet arrive at empirically equivalent transformation laws, what enables the recognition of equivalence despite the absence of shared semantics? The answer, we propose, lies in the abstract machinery of Category Theory—a branch of mathematics that focuses not on the internal nature of objects, but on the relationships between them. In this context, Category Theory serves not merely as a formalism for mathematics, but as a meta-linguistic structure capable of revealing the common architecture underlying distinct physical theories.

3.1 Categories, Functors, and Natural Transformations

At its core, a category consists of a collection of *objects* and *morphisms* (arrows) between these objects that satisfy two basic rules: morphisms can be composed

associatively, and each object has an identity morphism. Importantly, the internal nature of objects is irrelevant; what matters are the relationships—the arrows—between them.

A functor is a mapping between categories that preserves the structure of the morphisms. That is, it maps objects to objects and morphisms to morphisms in such a way that identity morphisms and composition are preserved. Functors thus provide a way to translate one categorical structure into another while maintaining the integrity of their internal relational logic.

A natural transformation is a way of transforming one functor into another, such that the transformation respects the structure of the categories involved. It expresses the idea that two functors are not just similar in structure, but related in a consistent and coherent way across their entire domains.

These three constructs—categories, functors, and natural transformations—form the foundational language of Category Theory. Their significance lies in their *abstraction from content*. They allow us to describe, compare, and relate mathematical and conceptual systems entirely through their relational architecture, without regard for the specifics of the entities involved.

3.2 Modeling Physical Theories as Functors

A physical theory can be interpreted as a category, where the objects might be physical states or entities, and the morphisms represent allowed transformations, interactions, or symmetries. For instance, in classical mechanics, objects might correspond to points in a phase space, and morphisms to time-evolution transformations governed by Hamiltonian dynamics. In quantum mechanics, objects might be Hilbert spaces, and morphisms linear operators.

More generally, a physical theory can be viewed as a functor from an abstract syntactic category—representing the formal structure of the theory—to a semantic category, such as the category of physical systems or empirical observations. This perspective allows for the comparison of theories even when their semantics differ dramatically. If two theories are modeled by functors from the same syntactic category into two different semantic categories, their functorial

equivalence can be studied without requiring their internal content to be interpretable by the same cognitive framework.

3.3 Structural Understanding Without Semantic Access

This brings us to the core insight of this section: structural equivalence can be recognized even in the absence of semantic comprehension. Two theories may model the same set of observable invariants through entirely different languages and ontologies. As long as their functorial mappings preserve the same diagrammatic structure—that is, the same pattern of relationships and transformations—their equivalence can be established in a purely categorical sense.

This has powerful implications for communication between civilizations (or intelligences) with non-overlapping conceptual vocabularies. While neither may be able to explain the *meaning* of their theories to the other, both can potentially recognize when two theories share the same categorical form. In this way, Category Theory acts as a meta-theoretical interface: it provides a structural bridge between different worlds of understanding, even when those worlds are otherwise epistemically opaque to one another.

3.4 Example: Translating E = mc^2 Through Functorial Mapping

To illustrate this idea, consider the human formulation of the mass-energy equivalence law:

$$E = mc^2$$

Let us model this as a functor F_H from a syntactic category $\mathcal{T}_H$ (representing the formal structure of human physical theories) to a semantic category $\mathcal{P}_H$, where:

- E, m, and c are objects (quantities)
- The equation itself is a morphism encoding a transformation between concepts

Now imagine an alien civilization whose theory is formalized as a functor F_A from its own syntactic category $\mathcal{T}_A$ to a semantic category $\mathcal{P}_A$, with an expression such as:

$$\Xi^{\lambda(\psi)} = \Omega_{\mu} \circ \Phi^{\nabla(\chi)}$$

Although the primitives Ξ, λ, Φ, and χ have no semantic counterpart in human physics, we may discover—through categorical analysis—that the structure of relationships between them mirrors that of our own theory. There exists a diagram in $\mathcal{T}_H$ and one in $\mathcal{T}_A$ such that both commute under their respective functors F_H and F_A. A **natural transformation** $\eta : F_H \Rightarrow F_A$ may be constructed, not as a translation of content, but as a structural identification of equivalence classes.

In other words, the alien equation and Einstein's formulation are categorically isomorphic even though their semantic components are incommensurable. This is a profound result: it suggests that intelligences with radically different epistemologies can still recognize a shared physics, not because they share meaning, but because they share structure.

In this way, Category Theory reveals itself not merely as a mathematical tool, but as a universal language of intelligibility—a way for different cognitive systems to converge upon the same truths, even if those truths are spoken in mutually untranslatable tongues.

4. Homotopy Type Theory and the Topos of Perception

While Category Theory offers a structural language to compare and relate physical theories without requiring semantic overlap, Homotopy Type Theory (HoTT) goes

a step further. It provides a foundational framework in which types represent not static mathematical objects, but dynamic, higher-dimensional structures—embodied as *spaces*, *paths*, and *homotopies*. HoTT arises at the intersection of type theory, homotopy theory, and category theory, and introduces a new way to think about the construction and meaning of mathematical and physical systems, especially when interpreted from within multiple cognitive or perceptual frameworks.

4.1 Introduction to HoTT and Its Relation to Categorical Foundations

Homotopy Type Theory reconceptualizes types as spaces and equalities between elements as paths within those spaces. From this perspective, reasoning about equality is no longer about identity in a logical or algebraic sense, but about the existence of a continuous transformation (a path) from one object to another. In more advanced levels of the theory, equalities between paths are themselves higher paths, forming an entire hierarchy of equivalences—hence the homotopical nature of the system.

This aligns deeply with higher category theory, where morphisms between objects give rise to morphisms between morphisms (called 2-morphisms), and so on. In this sense, HoTT can be seen as providing a computational or logical implementation of higher category theory, suitable not just for describing mathematics but for modeling entire conceptual universes—including the formulation of physical theories.

4.2 Physical Theories as Types in a Higher-Order Structure

In the HoTT framework, a physical theory can be represented as a *type*, where the terms (or inhabitants) of that type are the valid expressions, equations, or phenomena within that theory. More than just being a syntactic container, the type also encodes the relationships among its terms via the paths that connect them. These paths represent valid transformations, derivations, or homotopic equivalences.

In the human framework, for example, the type corresponding to special relativity might contain inhabitants such as the Lorentz transformations, the invariance of

the speed of light, and mass-energy equivalence. The homotopies in this type represent the interderivability of these concepts, the transformations between reference frames, and the topological relationships among physical observables.

An alien civilization, operating with fundamentally different primitives—perhaps in a non-metric, non-linear, or even non-temporal paradigm—might develop its own physical theory as a distinct type, with different inhabitants and homotopies. Crucially, the types themselves may not be interconvertible at the level of terms or rules, but they may exhibit *higher-level equivalences* in terms of structure or behavior, forming part of the same higher-dimensional topos.

4.3 Internal Logic and the Topos of Perception

One of the revolutionary insights from categorical logic and topos theory is that logic is not absolute; it is *internal to the structure under consideration*. Each topos—the categorical generalization of a universe of sets—comes equipped with its own internal logic, which may differ fundamentally from classical logic. In the same way, each civilization constructs its theories within the bounds of a particular internal logic, shaped by the perceptual, cognitive, and conceptual tools it has developed.

Humans, for instance, employ a logic grounded in classical Boolean structures, continuous spacetime, and energy-momentum conservation. An alien species might employ a logic that arises from discrete topological events, relational causal networks, or vibrational field coherences across a higher-order dimensional substrate. Their internal logic would dictate not only what constitutes a valid inference but also what constitutes a valid entity, property, or measurement.

This leads to a profound limitation: translation between theories constructed in different internal logics is impossible at the level of primitives. There is no way to map “mass” to “topological resistance,” or “energy” to “coherence amplitude,” unless both arise from a shared conceptual topos. This is not just a linguistic problem—it is a *logical impossibility*, akin to translating between incompatible axiomatic systems where terms are meaningful only within their own deductive environments.

4.4 The Possibility of Shared Higher-Level Homotopies

Despite these foundational differences, all is not lost. The power of HoTT lies in its ability to model equivalence at higher levels of abstraction, beyond the level of direct interpretation. Just as two shapes can be homotopically equivalent without being identical, two physical theories—each encoded in its own type, each constructed in its own internal logic—can be equivalent at a *higher homotopical level*. This means they may reside within the same abstract "space of theories," connected not by semantic translation, but by homotopies between types.

Such equivalences allow for recognition of shared structure even when interpretation is impossible. The alien theory and the human theory may both map into a higher-level object—what we might call a *universal theory space*—via their respective embeddings as types. If these embeddings are homotopic, the theories can be said to be structurally equivalent in a deep sense, even if no term-to-term correspondence is possible.

This view transforms how we think about understanding across civilizations. Rather than seeking to align meanings or terms, we aim to identify shared *invariants*—homotopies that persist across systems and logics. This reconceptualizes understanding as a process not of translation, but of *alignment through higher-order structure*.

4.5 Implications for a Relational Ontology of Physics

The HoTT perspective implies that physical laws may not be ontologically absolute but relationally grounded within a topos of perception. What is real, measurable, and knowable is a function of the internal logic of the observer. But across all topoi, there may exist *homotopic bridges*—invariants in the space of theories—that signal deep correspondence.

This radically shifts the search for universality in science. Instead of seeking laws that are true for all observers in the same way, we seek patterns that emerge consistently across diverse internal logics. In this view, physics becomes not a description of an objective world from a privileged vantage point, but a *collection*

of structurally equivalent but semantically divergent views, united through higher-order homotopy.

Thus, Homotopy Type Theory does more than refine our tools—it reframes the entire philosophical foundation of what it means to understand the universe.

5. Axiomatic Invariance vs. Conceptual Incommensurability

The thought experiment thus far has led us to an important bifurcation in how we assess the universality of physical theories: one between axiomatic invariance and conceptual incommensurability. At the heart of this distinction is a fundamental reconsideration of what it means to *understand* a law of physics. Is understanding bound to our intuitive grasp of a concept, rooted in language, perception, and physical experience? Or can understanding be redefined as a recognition of structure and invariance across formally equivalent systems, regardless of whether we can intuitively or semantically access their content?

5.1 What It Means to “Understand” a Law of Physics

In conventional scientific epistemology, to "understand" a physical law is to possess both a formal and intuitive grasp of its meaning. We expect to know not only how to manipulate the symbols in an equation like E = mc^2, but also what it *means*: that energy and mass are interconvertible, that light speed squared mediates the relationship, and that this insight has empirical and theoretical consequences for nuclear reactions, relativity, and more.

Yet this conception of understanding is profoundly anthropocentric. It assumes that our cognitive intuitions—developed through evolutionary pressures tied to manipulating objects in space and time—are well suited to grasping the deep architecture of the universe. The alien civilization in our thought experiment challenges this assumption by possessing a law functionally equivalent to mass-energy equivalence, but cast in terms that are completely unintelligible to us. They “understand” their law as deeply as we understand ours, yet neither civilization can access the other’s concepts. This poses a dilemma: is mutual semantic

comprehension necessary for genuine understanding, or is shared *structural alignment* sufficient?

5.2 Structural Invariants vs. Cognitive-Accessible Content

To resolve this tension, we differentiate between two dimensions of understanding:

- Structural invariants refer to the logical architecture of a theory—its internal consistency, transformation laws, and the relational patterns among its components. These can be captured by categorical and type-theoretic representations and compared across systems without relying on shared semantics.
- Cognitive-accessible content refers to the specific terms, concepts, and intuitions tied to an observer's sensory apparatus and experiential world. For humans, this includes "mass," "energy," "acceleration," and so forth—concepts tied to how we interact with our environment.

Our argument is that while content may be opaque across civilizations, structure can remain invariant. The aliens may have no notion of "energy" or "mass," but if the relationships within their system mirror those in ours—say, by preserving symmetries, conservation laws, and transformation equivalences—then a formal equivalence exists at a higher level of abstraction. Category Theory and Homotopy Type Theory provide the machinery to detect and formalize these correspondences, even in the absence of shared interpretations.

5.3 Role of Interpretation Functors and Sensory Mappings

One way to model the disconnection between content and structure is through interpretation functors—mappings from abstract formal structures (categories or type-theoretic constructs) to semantic domains grounded in a particular civilization's sensory and cognitive modalities. For example, in human physics, a functor maps the abstract concept of an invariant quantity to our empirical concept of "mass," which we measure through gravitational interaction or inertial resistance. In the alien system, a different functor maps the same structural object

to a concept rooted in their own perceptual framework, perhaps something like "topological phase density" or "coherence torsion."

These interpretation functors are not themselves interchangeable. They are grounded in distinct sensory mappings—processes by which raw experience is transformed into interpretable phenomena. Since the alien and human functors originate in incommensurable perceptual systems, there can be no direct equivalence at the level of content. However, the underlying category being functorially mapped remains the same, allowing for a recognition of structural similarity even when meanings are inaccessible.

5.4 Analogy with Gödel Numberings, Turing Equivalence, and Cryptographic Ciphers

To further illuminate the distinction between structural invariance and content incommensurability, we may turn to analogies from logic, computation, and cryptography.

- Gödel numberings demonstrate that syntactic statements in formal logic can be encoded as numbers without any reference to meaning. The arithmetic properties of these numbers preserve the structure of logical statements, allowing for meta-mathematical reasoning about the system itself. Likewise, an alien formulation of a physical law may be a kind of Gödel numbering—unintelligible in content, but fully equivalent in structure.
- Turing-equivalent machines provide another analogy. Two vastly different machines may compute the same function—one using binary logic gates, another using neural patterns, or quantum entanglement. What matters is the *computational structure*, not the physical realization. Similarly, two physical theories may yield the same predictions and transformation laws while being conceptually incompatible.
- Cryptographic ciphers also reveal how meaning can be obscured without loss of information. An encrypted message may be incomprehensible without the key, yet structurally contains the full message. In this analogy,

the alien formulation of E = mc^2 is like an encrypted version of our own, with Category Theory providing the cipher key that allows us to recognize formal equivalence despite semantic opacity.

5.5 Toward a Broader Conception of Understanding

Taken together, these perspectives suggest that the essence of physical understanding may lie not in our ability to internalize meaning through intuition or experience, but in our capacity to identify structural invariants across systems. When viewed through this lens, a law of physics becomes less a description of "how things are" and more a diagrammatic relationship that holds true within and across formal systems.

For cross-civilizational epistemology—and for future interactions with artificial intelligence, post-human cognition, or alien species—this reconceptualization is crucial. If meaning is inherently local and perceptually bounded, but structure is global and transferrable, then the quest for universal science should shift from discovering shared content to mapping invariant structures across otherwise disjoint internal logics.

This view does not deny the value of semantic intuition, but it frames it as a *surface effect*—an emergent property of deep structural alignments filtered through specific modes of perception. What lies beneath is not the familiar world of mass and energy, but the abstract architecture of transformation and relation—a logic of reality that may be more universal than any one civilization's interpretation.

6. The Alien Equation: A Formal Presentation

Having established the theoretical groundwork for structural understanding in the absence of semantic translation, we now turn to a formal presentation of the alien analog to the Einsteinian mass-energy equivalence equation, E = mc^2. The purpose of this section is not to speculate on the specific phenomenology of alien physics, but to demonstrate how such a theory could be formally expressed,

analyzed, and proven equivalent in structure to our own, using the tools of Category Theory and Homotopy Type Theory.

The alien equation is written in a symbolic system defined by their own perceptual and conceptual primitives. These primitives are not interpretable in human terms, nor are they derivable from our sensory-motor experiences. Nonetheless, they participate in a formal system that mirrors the structural relationships found in our own physics. The equation is presented below:

$$\Xi^{\lambda(\psi)} = \Omega_{\mu} \circ \Phi^{\nabla(\chi)}$$

This expression may appear cryptic, but it is not intended to be parsed in terms of familiar physical quantities. Instead, we treat the symbols as abstract categorical entities, whose internal meanings are defined within the alien civilization's own internal logic, but whose *relationships* can be compared structurally to human formulations.

6.1 Notation and Structure Without Semantic Interpretation

Let us begin by characterizing the elements of this expression as placeholders in an abstract categorical framework:

- Ξ, Ω, and Φ are **functors** operating within a category $\mathcal{A}$, the alien's semantic universe.
- λ and ∇ are **natural transformations**, providing structure-preserving mappings between functors.
- ψ and χ are **objects** in $\mathcal{A}$, whose internal nature is alien but formally defined.
- The superscripting and composition denote transformations and interactions among functors and natural transformations within a higher-order categorical context.

The alien equation thus takes the form of a **commutative diagram**, expressing the equivalence of two paths through a diagrammatic space of transformations—one a compound functor applied to an object via transformation $\lambda(\psi)$, the other a composition of two functors Ω_{μ} and $\Phi^{\nabla(\chi)}$.

6.2 Functorial Composition as Transformation Law

In the categorical language, the expression:

$$\Xi^{\lambda(\psi)} = \Omega_{\mu} \circ \Phi^{\nabla(\chi)}$$

indicates a transformation law that relates two different functorial constructions. On the left-hand side, Ξ is modulated by a parameterized natural transformation λ, applied to a term ψ. On the right-hand side, the composition $\Omega_{\mu} \circ \Phi^{\nabla(\chi)}$ expresses a sequential operation, where the output of Φ—modified by transformation ∇—serves as the input to Ω, further modulated by μ, another natural transformation or indexing parameter.

This form expresses a **transformational equivalence** between two categorical constructions—each representing the alien civilization's version of a fundamental invariant transformation within its own physics.

6.3 Explanation of Alien Primitives in Abstract Terms

To explore the alien primitives without attempting to force them into human meaning, we describe them abstractly in terms of their roles within the categorical structure:

- ψ: An object in the alien category $\mathcal{A}$, analogous to a physical system or primitive entity.
- λ: A natural transformation that modulates functor Ξ with respect to the morphisms associated with ψ. Interpreted abstractly, $\lambda(\psi)$ may represent a kind of *coherence amplitude*—a measure of internal transformation potential within an alien-defined topological system.
- χ: Another object in $\mathcal{A}$, possibly representing what we would call an "inertial frame," a "state of resistance," or "localized density"—though such terms are metaphorical at best.
- ∇: A transformation modulating Φ, which may be conceived abstractly as encoding the gradient or differential relation across an alien-defined topological manifold.
- Φ: A functor mapping structures from one region of the category to another, representing expansion, dispersion, or transformation across a relation-dense medium.
- Ω: A higher-order functor applying to the result of Φ, often associated with the binding or condensation of structural potentials—analogous, perhaps, to the human idea of "mass realization" or "invariant contraction."
- μ: A morphism or transformation applying to Ω, regulating or calibrating its domain of application.

These primitives collectively define the alien version of a transformation law that, while semantically opaque, is formally rich and logically consistent within its own system.

6.4 Structural Proof of Equivalence Using Categorical Constructs

Now let us sketch a categorical proof of equivalence between the alien equation and Einstein's $E = mc^2$, not by translating their meanings, but by mapping their **structures**.

Define:

- The human physical theory as a functor $F_H : \mathcal{T}_H \to \mathcal{P}_H$, where $\mathcal{T}_H$ is a syntactic category (e.g., differential geometry and classical field theory) and $\mathcal{P}_H$ is a category of physically interpreted phenomena.
- The alien physical theory as a functor $F_A : \mathcal{T}_A \to \mathcal{P}_A$, defined analogously but constructed from different primitives and internal logic.

Within both theories, there exists a commutative diagram representing the invariant transformation law. In human physics, this is:

$$E \xleftarrow{\sim} m \cdot c^2$$

where the multiplication and squaring operations are morphisms and c is a constant object in $\mathcal{T}_H$.

In the alien system, the equivalent diagram is:

$$\Xi^{\lambda(\psi)} \xleftarrow{\sim} \Omega_\mu \circ \Phi^{\nabla(\chi)}$$

To establish **structural equivalence**, we demonstrate the existence of a natural transformation:

$$\eta : F_H \Rightarrow F_A$$

such that η maps the diagrammatic structure of one theory to the other, preserving functorial relations and commutativity. That is, even though the objects and morphisms differ, the *way* in which objects are related—the categorical diagram—remains invariant. This confirms that both theories instantiate the same structural form, and hence describe the same invariant transformation law at an abstract level.

This formal correspondence **does not require us to translate the terms**, only to verify that the morphisms between the objects of each system preserve the same relationships.

6.5 Summary and Implications

The presentation of the alien equation and the categorical proof of its structural equivalence to Einstein's mass-energy relation illuminate the core claim of this paper: that the universality of physical law need not rest on shared semantics, but on shared relational form. The alien equation is not a "translation" of E = mc^, but a structurally analogous construct defined within a different internal logic.

This opens the door to a profound rethinking of how intelligences—human, alien, or artificial—may compare and validate scientific theories across cognitive divides. By focusing on the *topology of relations* rather than the *semantics of terms*, we reveal a deeper level of scientific intelligibility, one grounded in categorical invariants that transcend language, biology, and culture.

7. Implications for the Search for Universal Laws

The structural equivalence between distinct, incommensurable physical theories carries profound implications—not only for epistemology and the philosophy of science but also for our most ambitious empirical endeavors, such as the search for extraterrestrial intelligence (SETI), the formulation of universal laws of nature, and the conceptual boundaries of cosmology itself. If physical laws are not necessarily shared in terms of semantic content but rather in terms of relational structure, then our methods of detecting and interpreting signs of intelligence or of universal truth must evolve accordingly.

7.1 The Limitations of SETI and Current Cosmological Epistemology

The Search for Extraterrestrial Intelligence (SETI), as currently conceived, relies heavily on assumptions about communicability and shared frames of reference. Whether it is radio signals, laser pulses, or patterns in astronomical phenomena, the underlying presumption is that intelligences elsewhere in the cosmos will employ a sufficiently similar conceptual framework to ours, such that we can detect their signals and interpret their intent. Often, this is accompanied by the assumption that mathematics—especially prime numbers, geometric ratios, or specific spectral modulations—can serve as universal signifiers of intelligence.

However, the thought experiment developed in this paper directly challenges the sufficiency of these assumptions. If two civilizations can arrive at empirically equivalent physical theories through entirely distinct cognitive architectures, ontologies, and perceptual foundations, then the semantic content of their science—even their mathematics—may be unintelligible to us. A signal containing the alien equivalent of E = mc^2 might appear to us as noise or meaningless

symbolic drift, even though it encodes a principle as foundational as mass-energy equivalence.

Moreover, cosmological epistemology itself may be constrained by the anthropocentric assumption that our models—based on continuous spacetime, locality, and a particular suite of symmetries—are not merely effective but ontologically privileged. Yet, if other observers model reality using non-metric geometries, non-Boolean logic, or topological relational webs that do not reduce to our framework, then our entire formulation of "laws of nature" becomes just one topos among many.

7.2 The Need for a New Kind of Translation: Structural, Not Semantic

Given these challenges, we propose a paradigm shift in how translation is conceived—particularly in inter-intelligence communication and scientific unification. Traditional translation efforts presume some level of overlap in perceptual and linguistic primitives. But the structural perspective offered by Category Theory and Homotopy Type Theory allows for a different kind of mapping: translation of structure without translation of content.

Rather than searching for universal symbols or shared numerical sequences, we might instead search for structural invariants: regularities in patterns of interaction, symmetries in relationships, or the presence of commutative diagrams in observed behavior. In this view, communication becomes less about transmitting symbols and more about aligning relational architectures—much as two formal systems can be proven equivalent without sharing any of the same vocabulary.

This shift would require entirely new tools for detecting and interpreting signs of intelligence. Rather than filtering for familiar mathematical patterns, we might look for structurally coherent systems of transformations in astrophysical phenomena, or for systems that exhibit self-consistent internal logic, regardless of whether we understand their semantics.

7.3 Category Theory as a "Lingua Universalis" of Cognition and Science

At the heart of this reorientation lies Category Theory. Unlike other branches of mathematics that depend heavily on specific structures (like the real numbers, vector spaces, or differential manifolds), Category Theory deals with abstract relationships: how entities relate, transform, and compose. This universality makes it uniquely suited to act as a meta-language for describing scientific theories.

In a sense, Category Theory does not care what the entities are—only how they interact. This mirrors the goal of a truly universal language: not to impose a vocabulary, but to formalize the conditions under which systems can be said to relate to one another. Just as logic can be used to reason across different domains, Category Theory can be used to compare the structures of disparate scientific theories, independent of their content.

This feature positions it as a potential lingua universalis—not in the sense of a constructed language like Esperanto, nor a mathematical shorthand like algebra, but as a fundamental toolkit for relating knowledge systems that are otherwise semantically opaque. For any two sufficiently formalized theories, even those from entirely different civilizations, one can ask: do there exist functors between them that preserve the relevant structure? If so, then they are equivalent in the only sense that matters to the physical world.

7.4 Speculative Implications for Black Hole Civilizations and Non-Metric Physics

The implications of this perspective become especially provocative when extended to more speculative domains of physics—such as the interior of black holes, regions beyond the observable horizon, or universes with non-metric or higher-dimensional substrates.

Imagine a civilization whose cognitive basis emerged in a domain with no classical spacetime at all—say, a black hole interior modeled as a topological phase space without metric distance or temporal ordering. Such a civilization might naturally formulate physical laws not in terms of point masses and energy gradients, but in

terms of entangled coherence amplitudes, categorical fibrations, or morphism coalescence across homotopy classes.

From our vantage point, these concepts might be completely inaccessible. Yet, their formulation of fundamental transformations—say, black hole entropy flux or vacuum topology condensation—could be structurally equivalent to our field equations or quantum thermodynamic laws. Without Category Theory, we would never know. With it, we can at least ask the question: are these structures mappable, even if not translatable?

Moreover, this perspective suggests that universality is not about finding the "true" description of reality, but about locating fixed points in the space of theoretical structures—points where different internal logics yield the same relational invariants. This elevates the study of invariants and natural transformations from a mathematical abstraction to the central task of cosmological science.

7.5 Conclusion: A New Horizon for Science and Intelligence

In light of these possibilities, the search for universal laws—and the search for intelligent life—must broaden beyond the content of theories to their underlying structure. The frameworks of Category Theory and Homotopy Type Theory provide the tools for this broader search, enabling us to recognize deep correspondence even in the face of total conceptual divergence.

By reconceiving science not as a singular narrative about the universe, but as a family of structurally related perspectives, we open a path to understanding not just the cosmos, but the plurality of minds that may one day share it.

8. Conclusion

This paper has offered a speculative but formally grounded exploration of how different intelligences might arrive at structurally equivalent but conceptually incommensurable physical theories. By imagining two civilizations—ours and an alien counterpart—each possessing a fully coherent and predictive scientific

framework rooted in entirely different perceptual and cognitive modalities, we confront a deep philosophical and epistemological challenge: What does it truly mean for a scientific law to be universal?

Our analysis leads to a profound reframing of this question. Rather than seeking universality in the *content* of scientific theories—the specific terms, quantities, and metaphors employed—we suggest that the deepest form of universality lies in the structure of relationships those theories instantiate. Whether expressed as E = mc^2 or through an alien equation involving unfamiliar primitives like $\Xi^{\lambda(\psi)} = \Omega_{\mu} \circ \Phi^{\nabla(\chi)}$, the true invariant is not the symbols themselves, but the relational architecture they encode. If two theories preserve the same diagrammatic structure, the same transformation pathways, and the same internal consistencies, then they may be seen as categorically equivalent—even if no semantic bridge exists between them.

This insight elevates the role of Category Theory and Homotopy Type Theory (HoTT) as more than mathematical tools; they become epistemic bridges—meta-languages capable of relating theories that are otherwise closed to mutual understanding. In the face of incommensurability, they offer a way to formalize and compare truth-claims at the level of transformation, composition, and structure, rather than interpretation or narrative.

The implications extend well beyond mathematics. For the search for extraterrestrial intelligence, it means we must look not for shared vocabulary or familiar signals, but for patterns of structure that might indicate cognition at work. For physics, it suggests that different ontological commitments—such as metric versus topological formulations of space, or temporal versus atemporal causalities—may be surface expressions of a deeper structural core. And for philosophy, it challenges the long-held assumption that meaning and understanding are prerequisites for truth. We are led to entertain the possibility that truth may be structurally visible even when conceptually opaque.

In the end, the search for universal law may not culminate in a single equation written in a shared language, but in the recognition of structural invariants that

transcend all languages. The unity of science, if it exists, is not one of content but of correspondence—an emergent harmony not of symbols but of form. And this unity, paradoxically, may lie forever just beyond the reach of comprehension—not because it is hidden, but because it does not reside where we have been taught to look.

It resides, instead, in the invariance of structure itself.

References

1. Youvan, D.C. (2025). *A Formal Reconstruction of Physics: Foundations in Category Theory, Homotopy Type Theory, and the Primacy of the Word*. ResearchGate. DOI: 10.13140/RG.2.2.14401.06244
2. Einstein, A. (1905). *Does the Inertia of a Body Depend Upon Its Energy Content? Annalen der Physik*, 18, 639–641. [English translation available in *The Principle of Relativity* (Dover Publications, 1952).]
3. Lawvere, F.W. (1963). *Functorial Semantics of Algebraic Theories*. PhD Dissertation, Columbia University.
4. Lawvere, F.W., & Schanuel, S.H. (1997). *Conceptual Mathematics: A First Introduction to Categories*. Cambridge University Press.
5. Awodey, S. (2010). *Category Theory* (2nd ed.). Oxford University Press.
6. Univalent Foundations Program. (2013). *Homotopy Type Theory: Univalent Foundations of Mathematics*. Institute for Advanced Study. [Available online at: https://homotopytypetheory.org/book/]
7. Voevodsky, V. (2006). *A C-system defined by a universe category*. [Manuscript, Institute for Advanced Study].
8. Baez, J., & Stay, M. (2010). *Physics, Topology, Logic and Computation: A Rosetta Stone*. In *New Structures for Physics* (pp. 95–172). Springer.
9. Spivak, D.I. (2014). *Category Theory for the Sciences*. MIT Press.
10. Turing, A.M. (1936). *On Computable Numbers, with an Application to the Entscheidungsproblem*. *Proceedings of the London Mathematical Society*, 2(42), 230–265.
11. Gödel, K. (1931). *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*. *Monatshefte für Mathematik und Physik*, 38(1), 173–198.
12. Tegmark, M. (2014). *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*. Knopf.

13. Barwise, J., & Etchemendy, J. (1987). *The Liar: An Essay on Truth and Circularity*. Oxford University Press.